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AUTHOR	Kenworth, Lois
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ABSTRACT

This guidebook on minimum course content for beginning algebra covers graphing, the distance formula, slope, the slope-intercept form of the equation of a straight line, algebraic and graphic solutions to systems of equations, functions, and variation. Overall goals for the course are stated; performance objectives for each unit, a course outline, references to state-adopted texts, and teaching suggestions are given. A pretest and posttest are included, plus an annotated list of seven references. (DT)

AUTHORIZED COURSE OF INSTRUCTION FOR THE QUINMESTER PROGRAM

DADE COUNTY PUBLIC SCHOOLS

DIVISION OF INSTRUCTION • 1971

QUINMESTER MATHEMATICS

COURSE OF STUDY

FOR

ALGEBRA 1d

5215.24

(EXPERIMENTAL)

Written by

Lois Kenworth

for the

DIVISION OF INSTRUCTION
Dade County Public Schools
Miami, Florida 33132
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PREFACE

The following course of study has been designed to set a minimum standard for student performance after exposure to the material described and to specify sources which can be the basis for the planning of daily activities by the teacher. There has been no attempt to prescribe teaching strategies; those strategies listed are merely suggestions which have proved successful at some time for some class.

The course sequence is suggested as a guide; an individual teacher should feel free to rearrange the sequence whenever other alternatives seem more desirable. Since the course content represents a minimum, a teacher should feel free to add to the content specified.

Any comments and/or suggestions which will help to improve the existing curriculum will be appreciated. Please direct your remarks to the Consultant for Mathematics.

All courses of study have been edited by a subcommittee of the Mathematics Advisory Committee.

CATALOG DESCRIPTION

A study of graphing in the Cartesian plane and other related topics. Includes the distance formula, slope, the slope-intercept form of the equation of a straight line, algebraic and graphic solutions to systems of equations, functions, and work with variation.

Designed for the student who has mastered the concepts and skills of Algebra 1b.

TABLE OF CONTENTS

OVERALL GOALS

The student will:

1. Graph solution sets of open sentences in two variables.
2. Write equations of lines when given points on the lines and the slopes of the lines.
3. Solve systems of open sentences in two variables.
4. Begin to understand the concept of a function.
5. Further develop his ability to solve a variety of word problems.

KEY TO REFERENCES
(* State Adopted)

* D - Dolciani, Mary; Wooten, William; Beckenbach, Edwin; Jurgensen, Ray; and Donnelly, Alfred. Modern School Mathematics, Algebra 1, New York: Houghton Mifflin, 1967.

N - Nichols, Eugene D.. Modern Elementary Algebra. New York: Holt, Rinehart, and Winston, 1961.

*PL - Payne, Joseph; Zamboni, Floyd; and Lankford, Francis. Algebra One. New York: Harcourt, Brace and World, 1969.

*PA - Pearson, Helen R. and Allen, Frank B.. Modern Algebra: A Logical Approach, Book One. Boston: Ginn and Co., 1964.

[1] The number in the block preceding an objective indicates the number of the state assessment standard to which the objective is related.

PERFORMANCE OBJECTIVES

I. First-degree Sentences in Two Variables

The student will:

1. Graph a linear equation in two variables.

2. Graph linear inequalities in two variables.

3. Graph linear equations and inequalities involving absolute value.

4. Determine the slope of a line from its equation or graph.

5. Write a linear equation in slope-intercept form.

6. Write the equation of a line when given:
   a. the slope of the line and the y-intercept
   b. the slope of the line and the coordinates of one point on the line
   c. the coordinates of two points on the line.

7. Find the distance between two points by using the distance formula.

II. Systems of First-degree Sentences in Two Variables

The student will:

[6] 1. Solve a system of linear equations in two variables by graphing.

[6] 2. Solve linear systems by addition.

[6] 3. Solve linear systems by substitution.

[12] 4. Solve problems using a system of equations.

5. Solve a system of linear inequalities by graphing.

III. Relations and Functions

The student will:

[11] 1. Distinguish between a relation that is a function and one that is not a function.

[11] 2. Distinguish between the graph of a relation that is a function and one that is not a function.

3. Identify the domain and range of a relation or function.

4. Evaluate $f(x)$ for a given value of x and a given function.

5. Identify constant, linear, and quadratic functions from their graphs and from their rules.

6. Graph a quadratic function of the form $f(x) = ax^2 + c$.

7. Find the zeros of linear and quadratic functions algebraically and graphically (omit algebraic solution of quadratic functions if 1d is taken prior to 1c).

IV. Variation

The student will:

1. Identify proportions, direct variations, and inverse variations from their rules.

[12] 2. Solve problems involving proportions or direct or inverse variations.

COURSE OUTLINE

Related Objectives	
I.	I. First-degree Sentences in Two Variables
	A. Solutions by substitution 1. Linear equations 2. Inequalities
1,2,3	B. Solutions by graphing 1. Review of geometric sets (line, half-plane, etc.) 2. The coordinate plane a. correspondence between points of the plane and ordered pairs b. related terms - abscissa, ordinate, axis, origin, ordered pair, quadrant, coordinate 3. Graph ordered pairs 4. Give coordinates of a point 5. Graph linear equations 6. Graph equations involving absolute value 7. Graph inequalities
4,5,6,7	C. Linear Equations 1. Define slope 2. Find the slope a. given a graph b. given an equation written in slope - intercept form c. given two points on a line 3. Write the equation of a line a. given the slope and y-intercept b. given the slope and a point on the line c. given two points on the line 4. Graph an equation using the slope and y-intercept 5. Write equations of lines parallel and perpendicular to a given line 6. Find the distance between two points using the distance formula
II.	II. Systems of First-degree Sentences in Two Variables
1,2,3,4	A. Linear Equations 1. Solution set of the system as the intersection of the solution sets of the individual equations 2. Solution by graphing 3. Testing solutions by substitution

COURSE OUTLINE

Related Objectives		
II.		4. Types of Systems - recognition by graph or equation a. consistent b. inconsistent c. dependent d. independent 5. Equivalent systems a. definition b. identify from graph 6. Algebraic solutions a. by addition b. by substitution 7. Word problems
	5	B. Inequalities Solution by graphing
III.	III.	Relations and Functions
	1,2	A. Meaning 1. Definitions 2. Identification a. by ordered pairs b. by graph c. by rule
	3	B. Domain and range 1. Definitions 2. Finding a. from ordered pairs b. from graphs c. from rules
	4	C. Functional Notation 1. Reading and writing 2. Identifying the value of a function for a given argument a. from graphs b. from ordered pairs c. from rules 3. Evaluating functions given in functional notation
	5,6,7	D. Polynomial Functions 1. Definition 2. Linear 3. Constant

COURSE OUTLINE

Related Objectives		
		4. Identity 5. Quadratic a. solutions by substitution b. graphing c. identifying by graph or rule 6. Zeros of a function a. from graph b. algebraically (algebraic solutions to quadratics must be omitted if the student has not taken Algebra 1c)
IV.	IV.	Variation
1,2		A. Direct 1. Identify by rule or graph 2. Solve problems using direct variation 3. Set up proportions between pairs of a direct variation 4. Solve problems using proportions B. Inverse 1. Identify by rule or graph 2. Solve problems involving inverse variation

TEXTBOOK REFERENCES

Course Outline	D	N	PL	PA
I	189-216 434-436	259-268 276-281 393-398 410-412	162-196 205-210 289-291	428-464 487-490
II	223-253	268-276 281-284 293-307	220-246	465-486 490-492
III	377-388 394-403	398-404 408-410 413-415 (constant function and zeros of a function not covered)	174-185 350-359 (functional notation not covered)	542-561 588-590
IV	389-394 403-408	Variation not covered	196-204 (inverse variation not covered)	561-569

SUGGESTED STRATEGIES
(Keyed to Course Outline)

I. 1. Have the student find solutions to linear sentences in two variables by substitution. By finding many sets of ordered pairs that make the sentence true, the multiplicity of solutions becomes evident. Expanding this concept will help in developing the ideas of graphing on a plane.

2. Have the student test to see if given ordered pairs belong to the solution set of a certain equation or inequality. This helps enforce the idea that the graph is the set of all points that satisfy an equation.

3. Be sure that students have complete mastery of locating points on a coordinate plane before going into more complex graphing.

4. To facilitate graphing, have the student solve equations for the dependent variable. This not only makes graphing easier, but provides good review of the solution of an equation for one of its variables.

5. Since any number may be chosen for one of the variables, and then the value of the other variable determined, stress that the number chosen should be one that is easy to work with.

6. Stress the one-to-one correspondence that exists between ordered pairs of real numbers and points on a plane. This can be extended to ordered triples and space, if time and student interest permit.

7. The idea of a function machine is a help to students in understanding equations in two variables. A number is selected (the input) then the machine works on it to produce a second number the out put). Many students will be familiar with this idea from previous work and should be able to apply it readily to the present situation.

8. Have students draw graphs of equations other than linear equations by substituting values and then sketching a curve through points they have plotted. It not only illustrates different types of graphs, but is excellent review in evaluating polynomials.

9. The concept of a Cartesian product of two sets can be used to introduce graphing and also to introduce relations and functions.

10. Use set-builder notation whenever possible so the student feels comfortable with it.

11. The delta symbol may be introduced when defining slope, $m = \frac{\Delta y}{\Delta x}$. This could be beneficial for those students who will continue their mathematics studies.

12. When graphing inequalities, have the student rewrite the inequality as an equation, find the graph of the equation, and draw the graph as a line or dotted line depending on whether the inequality is $\geq$, $\leq$ or $>$, $<$. Then it is necessary to locate only one more point to determine the half plane which provides the solution set. Be sure to point out the advantages of testing another point as a check. This can be done by taking the coordinates of any point in the half plane and substituting them in the inequality to see if a true statement results.

II. 1. Introduce simultaneous equations by having the student graph two equations and find the coordinates of their point of intersection.

2. Define equivalent systems and use this definition in demonstrating the meaning of a graphical solution of simultaneous equations.

3. One method of stressing that the solution of simultaneous equations must satisfy each of the equations, is to give the student ordered pairs which are to be tested to determine if any one of them is the solution of a given set of equations.

4. Stress the definition of equivalent sentences and its use in solving simultaneous equations by the addition method.

5. Prepared overlay transparencies are excellent for introducing the graphing of compound inequalities.

6. Have students graph pairs of equations including parallels, perpendiculars, and equivalent equations. Use the graphs to introduce and discuss the terms consistent, inconsistent, dependent, and independent.

7. Have students make a chart showing the relationship between the coefficients of two equations, $AX + BY + C = 0$ and $DX + EY + F = 0$, and the kind of simultaneous equations that result - consistent and independent, inconsistent, or consistent and dependent.

8. Linear Programming is an excellent topic for enrichment and for showing an application of systems of equations.

9. The rationale for the addition method of solving systems of equations can be given as follows:

   If $AX + BY + C = 0$, then $m(AX + BY + C) = 0$ and if $DX + EY + F = 0$, then $k(DX + EY + F) = 0$, by the property of zero multiplication. $M(AX + BY + C) + k(DX + EY + F) = 0$, by the additive property of zero.

10. Simple determinants can be introduced for enrichment, then used to solve systems of equations.

11. Solution of three equations in three unknowns can be presented for the more able students.

III. 1. Introduce and use functional notation. It is not only good background for future mathematical work, but provides an excellent opportunity to review and practice computational skills.

2. Stress the difference between a relation that is a function and one that is not. This can be done by using graphs and testing whether any vertical line intersects the graph in more than one point.

3. If the function machine has not been introduced previously, then this section would be another good place for its introduction. If the student is familiar with the function machine then more work with it should help to clarify the concept of function.

4. A relation or a function may be defined by a set of ordered pairs, by a graph, by a rule, or by a mapping. It may be profitable to introduce all of the definitions to help clarify the concepts of relation and function.

5. Direct variation can be introduced as a special kind of linear function. Word problems can include many practical applications, the circumference of a circle varies directly as its radius, income from investment varies as the rate, area of a square varies as the square of the side, etc.

6. The section on quadratic functions allows for a good review if quin 1c has been taken prior to this quin. If this quin is taken before 1c, then there is an opportunity to show the need for learning to solve quadratic equations algebraically as well as graphically.

SAMPLE PRETEST ITEMS

This test covers some of the skills and concepts which are prerequisite to the development of topics in this quin.

Evaluate each of the expressions for the given values of the variables:

1. $x^2 - 5x + 3$ $\quad x = -2$
2. $a - |-b + a|$ $\quad a = 2, b = -5$
3. $6y - y^2 + 3$ $\quad y = \frac{3}{4}$
4. $x^2 + |2x - 12|$ $\quad x = 3$
5. $5 - 7c + 2c^2$ $\quad c = 4$

Graph the solution set of each of the following open sentences using the set of real numbers as the replacement set:

6. $6 - 2x = 22$
7. $4x < 12$
8. $5 - y \geq 3$
9. $-4 < 6 - 2x \leq 12$

Solve for N:

10. $\frac{5}{N} = \frac{12}{19}$
11. $\frac{6}{23} = \frac{39}{N}$

Multiply:

12. $3(2x - 7y + 4)$
13. $-5(3a + 4b - 6)$

Add:

14. $4x - 7y$ and $2y - 9x$
15. $3a + 2b + 4$
    $4a - 2b - 9$

SAMPLE PRETEST ITEMS

Subtract:

16. $8x - 3y + 7$
$-2x - 3y + 4$

17. $4y + 2d - c$
$4y - 2d + c$

18. $3m + 2n + 7$
$-5m + 4n - 3$

Write complete solutions for the following problems:

19. The sum of two consecutive odd numbers is 48. What are the numbers?

20. A freight train leaves Boston at 9 a.m., traveling at 30 miles per hour. How fast is a passenger train traveling if it leaves the same station at 10:30 a.m. and overtakes the freight train at 12:30 p.m.?

KEY TO PRETEST

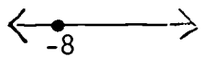
1. 17

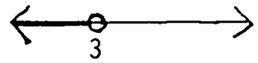
2. -5

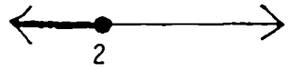
3. $\frac{111}{16}$

4. 15

5. 2.52

6. 

7. 

8. 

9. 

10. $7\frac{11}{12}$

11. $149\frac{1}{2}$

12. $6x - 21y + 12$

13. $-15a - 20b + 30$

14. $-5x - 5y$

15. $7a - 5$

16. $10x + 3$

17. $4d - 2c$

18. $8m - 2n + 10$

19. Let x = one number
Then x + 2 = the second number

$$x + x + 2 = 48$$
$$2x + 2 = 48$$
$$2x = 46$$
$$x = 23$$

$$x + 2 = 25$$

20. Let x - rate of passenger train in miles/hour.

$$2x = (2 + 1\tfrac{1}{2})\ 30$$

$$2x = \tfrac{7}{2} \cdot 30$$

$$2x = 105$$

$$x = 52\tfrac{1}{2} \text{ miles per hour}$$

SAMPLE POSTTEST ITEMS
(Keyed to Objectives)

I. Graph each of the following open sentences.

1. a. $x + y = 3$

   b. $x - 2y = 5$

2. a. $2x + 3y > 6$

   b. $x - 2y \leq 4$

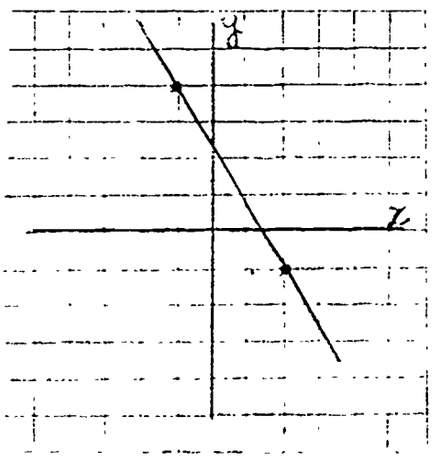
3. a. $y = |x| - 5$

   b. $|3x| - 2y \geq 12$

4. Find the slope of the line.

   a. $4y + x = 7$

   b.



5. Write the following equations in slope-intercept form.

   a. $2y = 6x - 8$

   b. $5y - 2 = 10x$

6. Write the equations, with integral coefficients, of the lines described below.

   a. A line whose slope is $-\frac{1}{2}$ and y-intercept is 3.

   b. A line whose slope is $\frac{5}{4}$ and that passes through (-2,2).

   c. A line that passes through (5,0) and (-1,3).

7. Find the distance between the points $(\frac{1}{2}, -1)$ and $(\frac{1}{2}, 4)$

II. 1. Solve the following system of equations by graphing. Write the solution set as an ordered pair.

$$\begin{cases} x - 2y = 0 \\ 4x + 2y = -10 \end{cases}$$

2. Solve the following systems by the addition method. Show all work.

a. $\begin{cases} x + y = 3 \\ 2x + y = 6 \end{cases}$

b. $\begin{cases} 5p - 2q = 11 \\ 3p + 5q = 19 \end{cases}$

3. Solve the following systems by the substitution method. Show all work.

$\begin{cases} x - y = 4 \\ 2x - y \end{cases}$

b. $\begin{cases} 2x - y = 7 \\ x + 4y = 2 \end{cases}$

4. Write a complete solution for each of the following problems using a system of equations.

a. A rectangle has a perimeter of 52 inches. The length of the rectangle is 7 inches less than twice its width. Find the length and width of the rectangle.

b. Two years ago, Beth was twice as old as David was then. In six years, Beth will be three times as old as David is now. How old are Beth and David now?

5. Solve graphically.

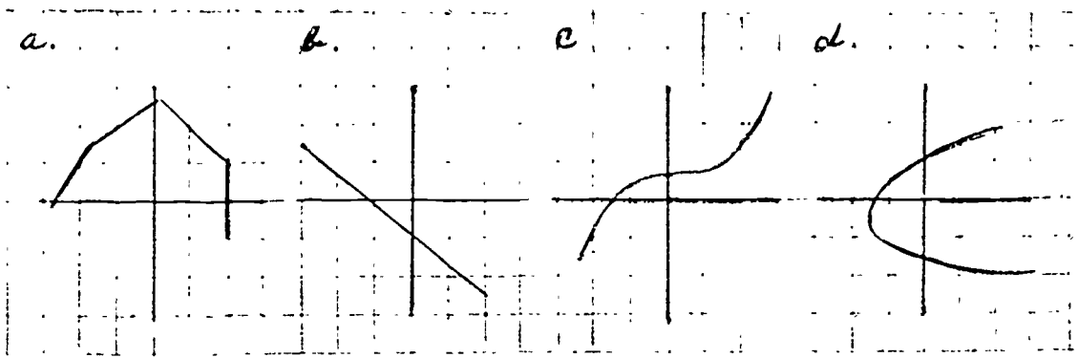
$$\begin{cases} x - y > -3 \\ 2x + y \leq 5 \end{cases}$$

III. 1. Tell whether each relation is a function.

a. $\{ (0, -2) (1, 5), (-3,7), (2, -2), (-4, -1) \}$

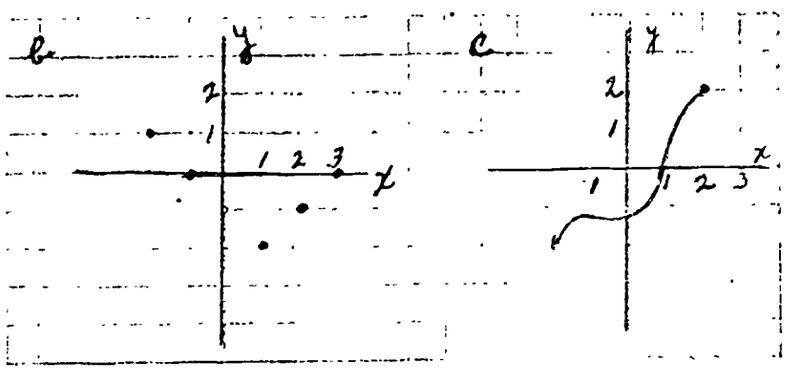
b. $\{ (3,7), (2,6), (1,5), (0,4), (0,3), (-1,2), (-2,1) \}$

2. Tell whether the graph is the graph of a function.



3. Find the domain and range of each of the functions.

   a. (0,1) (-6,3) (4,2)



4. a. Find $f(2)$ if $f(x) = -6x^2 + 3x - 4$.

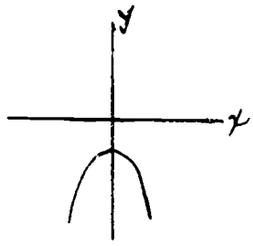
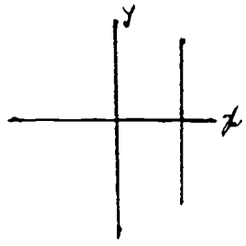
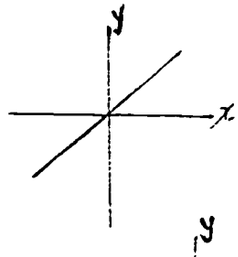
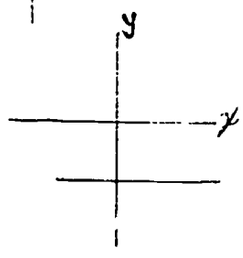
   b. Find $f\left(-\frac{1}{2}\right)$ if $f(x) = -3x^2 + 8$.

5. Match one item from column A and one item from column B with each of the given types of functions.

   a. constant function

   b. linear function

   c. quadratic function

Column A	Column B
m. $2x = y$	w. 
n. $x^2 - 2y = 0$	
o. $y = 5$	x. 
p. $y^2 + 2 = x^2 + 4x$	y. 
	z. 

6. Graph $y = 2x^2 - 4$

7. a. Find the zeros of the given functions algebraically.

   1 $2x - 3y = 7$

   2 $y = 3x^2 - 22x + 7$

   b. Find the zeros of the given functions graphically.

   1 $y = -\frac{2x}{3} - 2$

   2 $y = x^2 - 2x - 3$

IV. 1. Select from Column B those equations which are examples of the items in Column A.

Column A	Column B
a. Proportion	m. $y = kx$
b. Direct variation	n. $\frac{y}{x} = \frac{a}{b}$
c. Inverse variation	o. $y = x^2 + k$
	p. $y = \frac{k}{x}$
	q. $y + k = x$

2. Solve the following problems:

a. If y varies inversely as x and y = 4 when x = 7, find x when y = 14.

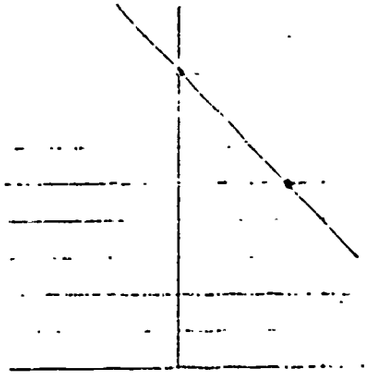
b. If 4 lbs. of apples cost $1.36, find the cost of 7 lbs.

c. If y varies directly as x, and y is -2 when x is 6, find x when y is 4.

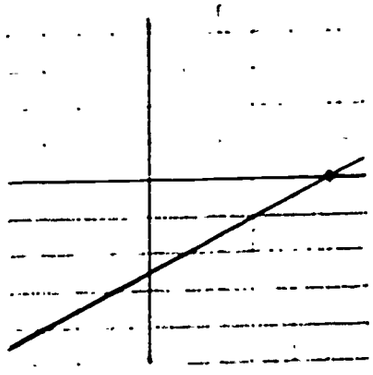
KEY TO POSTTEST

I.

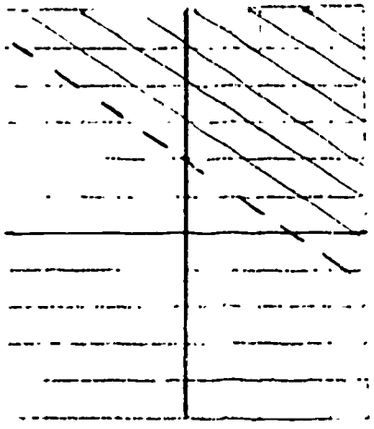
1. a.



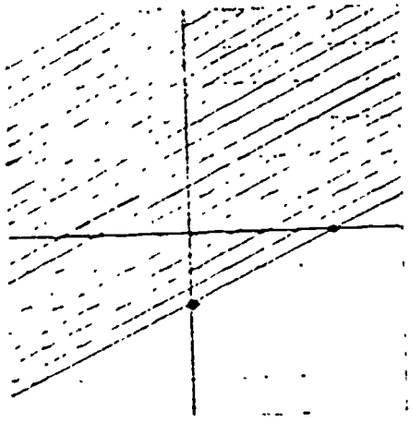
b.



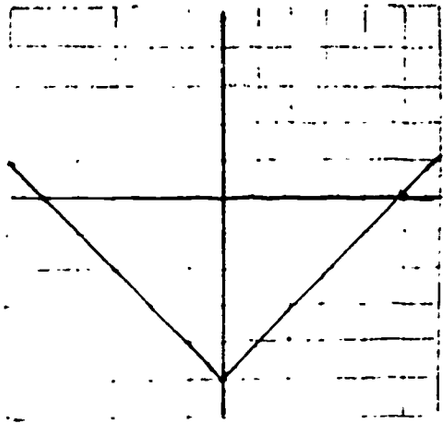
2. a.



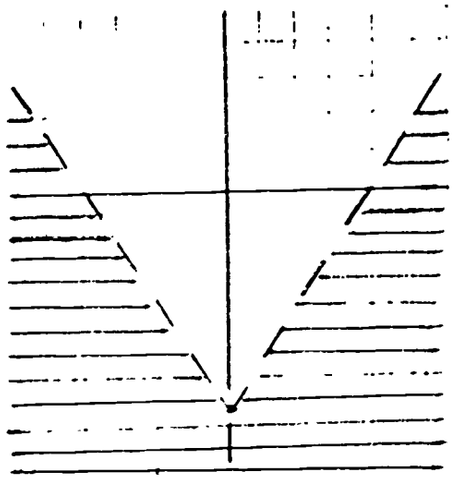
b.



3. a.



b.



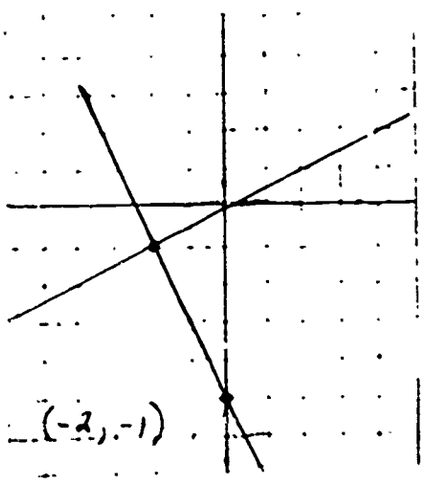
4. a. $M = -\frac{1}{4}$ b. $M = -\frac{5}{3}$

5. a. $y = 3x - 4$ b. $y = 2x + \frac{2}{5}$

6. a. $2y + x = 6$ b. $4y = 5x + 18$ c. $2y + x = 5$

7. 5

II. 1.



2. a.
$$\begin{array}{r} x + y = 3 \\ \underline{2x + y = 6} \\ x = 3 \end{array}$$

$3 + y = 3$
$y = 0$ (3,0)

b.
$$\begin{array}{r} 5p - 2q = 11 \\ 3p + 5q = 19 \end{array}$$

$$\begin{array}{r} 25p - 10q = 55 \\ \underline{6p + 10q = 38} \\ 31p = 93 \\ p = 3 \end{array}$$

$15 - 2q = 11$
$-2q = -4$
$q = 2$ (3,2)

3. a.
$$\begin{array}{r} x - y = 4 \\ 2x = y \\ \hline x - (2x) = 4 \end{array}$$

$-x = 4$
$x = -4$

$y = 2 \cdot (-4)$
$y = -8$ (-4, -8)

b. $2x - y = 7$
$x + 4y = 2$

$x = 2 - 4y$
$2(2 - 4y) - y = 7$
$4 - 8y - y = 7$
$-9y = 3$
$y = -\frac{1}{3}$

$2x + \frac{1}{3} = 7$
$x = 3\frac{1}{3}$ $(3\frac{1}{3}, -\frac{1}{3})$

4. a. Let x = width of rectangle = 11 inches
and y = length of rectangle = 15 inches

$$2x + 2y = 52$$

$$y = 2x - 7$$

$$2x + 2(2x - 7) = 52$$

$$2x + 4x - 14 = 52$$

$$6x = 66$$

$$x = 11$$

$$y = 2 \cdot 11 - 7$$

$$y = 15$$

b. Let x = Beth's age now = 6 years
and y = David's age now = 4 years

$$x - 2 = 2(y - 2)$$

$$x + 6 = 3y$$

$$x = 3y - 6$$

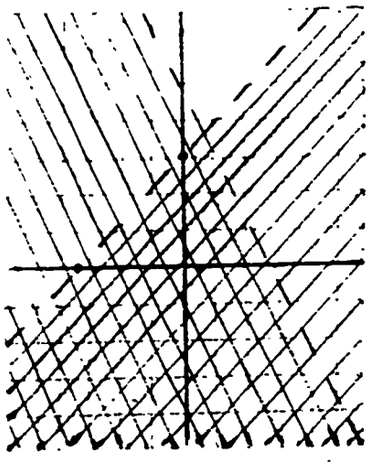
$$(3y - 6) - 2 = 2y - 4$$

$$3y - 8 = 2y - 4$$

$$y = 4$$

$$x = 3\ 4 - 6$$

$$x = 6$$

5. 

III. 1. a. function
b. not a function

2. a. not a function
b. function
c. function
d. not a function

3. a. domain $\{0, -6, 4\}$, range $\{1, 3, 2\}$

b. domain $\{-2, -1, 0, 1, 2, 3\}$, range $\{-2, -1, 0, 1\}$

c. domain $\{$Reals -2 to 2 inclusive$\}$

range $\{$Reals -2 to 2 inclusive$\}$

4. a. $f(2) = -22$

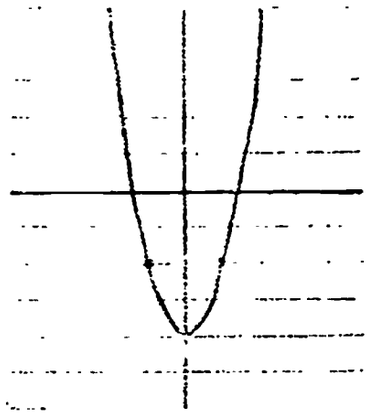
b. $f\left(-\frac{1}{2}\right) = 7\frac{1}{4}$

5. a. o and z

b. m and y

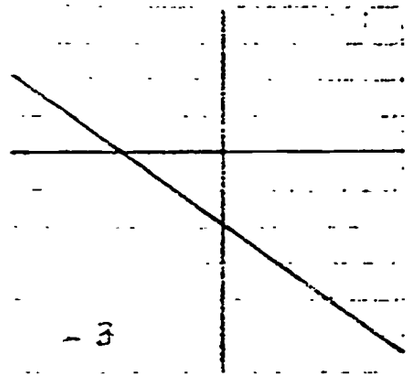
c. n and w

5.

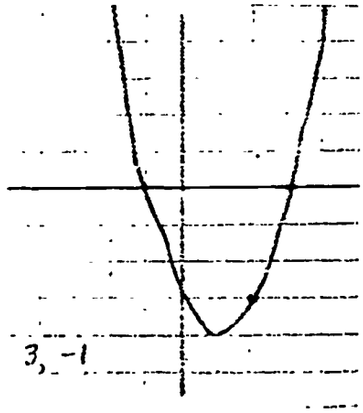


7. a. ① $\frac{7}{2}$ ② $\left\{\frac{1}{3}, 7\right\}$

b. ①



②



IV. 1. a. n
b. m
c. n

2. a. 2
b. $2.38
c. -12

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Contains 25 sets of linear equations which make pictures when graphed. Good for interest and practice in graphing.

Dressler, Isidore. Ninth Year Mathematics. New York: Amsco School Publications, Inc., 1966.

Pages 360-440 covers graphing of linear equations, proportions, and variation in review form with many appropriate exercises.

Heimer, Ralph T.; Kocher, Frank; and Luttes, John J.. A Program in Contemporary Algebra. New York: Holt, Rinehart and Winston, Inc., 1963.

Book 3 of this programmed series covers equations and inequalities in two variables. The last 2 units of Book 4 cover functions and relations.

Drooyan, Irving and Wooton, William. Programmed Beginning Algebra. New York: John Wiley and Sons, Inc., 1963.

This is the sixth book in an eight book sequence that covers the material of Algebra 1. Would be useful for students who have been absent or students who need additional work with graphing. Covers graphing of linear equations and inequalities, solving of simultaneous equations, and word problems.

Jacobs, Russell F. Introductory Algebra 2. New York: Harcourt, Brace and World, Inc., 1969.

Excellent development for the slower student or the student working on his own in the areas of relations, functions, solutions of simultaneous linear equations, graphing of linear functions, quadratic functions, and linear inequalities.

Peters, Max and Schaaf, William L. Algebra, A Modern Approach, Book 1. Princeton, New Jersey: D. Van Nostrand Company, Inc., 1968.

Includes a good section on direct and inverse variation which contains a number of work problems.

Fitzgerald, William M.; Zetterberg, Jack P.; and Dalton, LeRoy C.. Algebra 1, Theory and Application. River Forest, Illinois: Laidlaw Brothers Publishers, 1967.

Graphing is introduced through development of the concept of Cartesian products. First work in graphing is presented with the Cartesian produce I X I, and later expanded to Q X Q.